Introduction: When people process information manually, they deal with alphabetic and numeric characters. But when we process information with computer by typing some letters or words, the computer translate them in to binary numbers because digital computer is binary machine. The binary number system is the most important one in digital systems, but several others are also important. The decimal system is important because it is universally used to represent quantities outside a digital system. This means that there will be situations where decimal values must be converted to binary values before they are entered into the digital system.

Most commonly use number system:
A value of each digit in a number can be determined using (i) The digit, (ii) The position of the digit in the number, and (iii) The base of the number system. Where, base is defined as the total number of digits available in the number system.
So, different number systems have different base. There are several number systems in the world. But most commonly used number systems are (i) Decimal Number System, (ii) Binary Number System, (iii)Octal Number System and (iv) Hexadecimal Number System.

Decimal Number System:

For Integer Number
The number system that we use in our day-to-day life is the decimal number system. Decimal number system has base 10 as it uses 10 digits from 0 to 9. In decimal number system, the successive positions to the left of the decimal point represent units, tens, hundreds, thousands and so on i.e. Each digit position has a weight that is ten times the weight of the position to its right. Each position represents a specific power of the base 10.
For example, the decimal number 4587 consists of the digit 7 in the unit's position, 8 in the ten's position, 5 in the hundred's position, and 4 the thousand's position and its value can be written as:
$4 \times 10^3 + 5 \times 10^2 + 8 \times 10^1 + 7 \times 10^0$
$= 4 \times 1000 + 5 \times 100 + 8 \times 10 + 7 \times 1 = 4000 + 500 + 80 + 7 = 4587.$

You do in the same way $(785)_{10}$, $(2045)_{10}$, $(1210)_{10}$

For Fractional Decimal Number:

$$0.6321 = \frac{6}{10} + \frac{3}{100} + \frac{2}{1000} + \frac{1}{10000}$$
$$= 6 \times 10^{-1} + 3 \times 10^{-2} + 2 \times 10^{-3} + 1 \times 10^{-4}$$

So for fractional number, position is starting from 1 instead of 0 and starts form left to right with negative power. Therefore, for fractional number positional value will be power of -1, -2, -3, and so on, to its base.

Note: Convert integer and fraction parts separately and add together.

Binary Number System:
The binary number system is also a positional notation numbering system, but in this case, the base is not ten, but is two. Each digit position in a binary number represents a power of two. So, when we write a binary number, each binary digit is multiplied by an appropriate power of 2 based on the position in the number:
For example:
$(101101)_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$= 1 \times 32 + 0 \times 16 + 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1$
$= 32 + 8 + 4 + 1 = (45)_{10}$

Binary to Decimal Conversion:

For Integer Number
Any binary number can be converted to its decimal equivalent simply by summing together the weights of the various positions in the binary number that contain a 1.
For example: convert 101102 to decimal.

1 0 1 1 0
\ \ ___________ $1 \times 2^1 = 2$
\ _____________ $1 \times 2^2 = 4$
_______________ $1 \times 2^4 = \underline{16}$
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You do: convert 110112 to decimal. Ans. is 27.

For Fraction Number

In a binary fraction, the position of each digit (bit) indicates it s relative weight as was the case with the integer part, except the weights to in the reverse direction. Thus after the decimal point, the first digit (bit) has a weight of ½, the next one has a weight of ¼ , followed by 1/8 and so on.

Therefore, any fractional binary number can be converted by summing together the weights of the various fractional positions in the binary number that contain a 1.

For example, convert $(0.101)_2$ to decimal number.

$(0.101)_2 = 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$

$= \frac{1}{2} + \frac{0}{4} + \frac{1}{8}$

$= 0.5 + 0 + 0.125$

$= (0.625)_{10}$

You do: $(0.1101)_2 = (?)_{10}$ Ans. $= (0.8125)_{10}$

Decimal to Binary Conversion:

For Integer Number

There are two ways to convert a decimal *whole* number to its equivalent binary-system representation. **The first method** is the reverse of the process of decimal to binary conversion. The decimal number is simply expressed as a sum of powers of 2, and then 1s and 0s are written in the appropriate bit positions.

For Example: $45_{10} = 32 + 8 + 4 + 1 = 2^5 + 0 + 2^3 + 2^2 + 0 + 2^0$

$= (1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1)_2$

The **Second method** is repeated division method. It involves using successive division by the base i.e. 2 until the dividend/quotient reaches 0. At each division, the remainder provides a digit of the converted number, starting with the least significant digit (LSD) and the last remainder is the most significant digit (MSD). The binary number is formed with the help of remainder numbers starting from MSD to LSD.

An example of the process: convert 3710 to binary

Number/Base	**Quotient**	**Remainder**
37 / 2	18	1 (least significant digit)
18 / 2	9	0
9 / 2	4	1
4 / 2	2	0
2 / 2	1	0
1 / 2	0	1 (most significant digit)

The resulting binary number is: 100101 i.e. most significant digit to least significant digit.

You do: convert 93_{10} to binary. Ans. is 1011101.

For Fraction Number

To convert any fractional decimal number to binary, multiply the number with the base of binary number successively and record the integer parts, moving away from the decimal point to the right.

For example, convert $(0.625)_{10}$ to binary.

Number	Base	Product	Integer	Fraction
0.625	2	1.25	1	0.2500
0.2500	2	0.5	0	0.5000
0.5000	2	1	1	0.0000

Therefore, $(0.625)_{10} = (0.101)_2$

So putting together the integer and fractional part from the above we can say that $(3710.625)_{10} = (100101.101)_2$.

Octal Number System:

The octal number system is also a positional notation numbering system, but in this case base is eight. That means in octal number system there are eight numbers starting from 0 to 7. Each digit position in an octal number represents a power of eight. So, when we write an octal number, each octal digit is multiplied by an appropriate power of 8 based on the position in the number.

The reason for the common use of octal numbers is the relationship between the numbers 2 and 8. Eight is a power of 2 ($8 = 2^3$). Because of this relationship, three digits in a binary number can be represented with a single octal digit. This makes conversion between binary and octal numbers very easy and octal can be used to write large binary numbers with much fewer digits.

Octal to Decimal Conversion:

For Integer Number

Any octal integer number can be converted to its decimal equivalent simply by multiplying each octal digit by its corresponding place value and summarise them together.

For example:

$(2167)_8 = 2 \text{ x } 8^3 + 1 \text{ x } 8^2 + 6 \text{ x } 8^1 + 7 \text{ x } 8^0$

$= 2 \text{ x } 512 + 1 \text{ x } 64 + 6 \text{ x } 8 + 7 \text{ x } 1 = 1024 + 64 + 48 + 7 = (1143)_{10}$

For Fraction Number

Any fractional octal number can be converted as same way as above by summing together the weights of the various fractional positions in the octal numbers.

For example, convert $(0.763)_8$ to decimal number.

$(0.763)_8 = 7 \text{ x } 8^{-1} + 6 \text{ x } 8^{-2} + 3 \text{ x } 8^{-3}$

$$= \frac{7}{8} + \frac{6}{64} + \frac{3}{512}$$

$$= \frac{499}{512}$$

$= (0.9746)_{10}$

So summing together the integer and fraction part we can say that $(2167.763)_8 = (1143.9746)_{10}$

You do: $(0.248)_8 = (?)_{10}$

Decimal to Octal Conversion:

For Integer Number

To convert any decimal integer number to its octal equivalent you have divide the number by the base i.e. 8 and save the remainder until the dividend/quotient reaches 0. At each division, the remainder provides a digit of the converted number, starting with the least significant digit (LSD) and the last remainder is the most significant digit (MSD). The octal number is formed with the help of remainder numbers starting from MSD to LSD.

For Example: Convert $(1908)_{10}$ to octal.

Number/Base	Quotient	Remainder
1980/8	238	4 LSD
238/8	29	6
29/8	3	5
8/3	0	3 MSD

Therefore, $(1908)_{10} = (3564)_8$

For Fraction Number

To convert any fractional decimal number to octal number, multiply the number with the base of octal number (i.e. 8) successively and record the integer parts, moving away from the decimal point to the right.

For example, convert $(0.682)_{10}$ to octal.

Number	Base	Product	Integer	Fraction
0.6820	8	5.456	5	0.4560
0.4560	8	3.648	3	0.6480
0.6480	8	5.184	5	0.1840
0.1840	8	1.472	1	0.4720
0.4720	8	3.776	3	0.7760

So the $(0.682)_{10} = (0.53513.....)_8$ or we can write $(0.682)_{10} = (0.5351)_8$

Therefore from the above discussion we can write that $(1908.682)_{10} = (3564.5351)_8$

You do: $(285.28)_{10} = (?)_8$ Ans. $= (435.2172)_8$

Hexadecimal Number System:
Another number base that is commonly used in digital systems is base 16. This number system is called hexadecimal, and each digit position represents a power of 16. For any number base greater than ten, a problem occurs because there are more than ten symbols needed to represent the numerals for that number base. It is customary in these cases to use the ten decimal numerals followed by the letters of the alphabet beginning with A to provide the needed numerals. Since the hexadecimal system is base 16, there are sixteen numerals required. The following are the hexadecimal numerals:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
The reason for the common use of hexadecimal numbers is the relationship between the numbers 2 and 16. Sixteen is a power of 2 ($16 = 2^4$). Because of this relationship, four digits in a binary number can be represented with a single hexadecimal digit. This makes conversion between binary and hexadecimal numbers very easy, and hexadecimal can be used to write large binary numbers with much fewer digits.

Hexadecimal to Decimal Conversion:

For Integer Number
Any Hexadecimal number can also be converted to decimal number in the same way as binary to decimal or octal to decimal number system. That means in this case also multiplying each hexadecimal digit by its corresponding place value and summarise them together.
For Example: Convert $(3FA)_{16}$ to decimal equivalent.
$(3FA)_{16} = 3 \times 16^2 + 15 \times 16^1 + 10 \times 16^0$ [As F = 15 and A = 10]
$= 3 \times 256 + 15 \times 16 + 10 \times 1 = 768 + 240 + 10 = (1018)_{10}$

For Fraction Number
Any fractional h hexadecimal number can be converted to decimal number also by summing together the weights of the various fractional positions in the hexadecimal numbers.
For example, convert $(0.1A5)_{16}$ to decimal number.
$(0.1A5)_{16} = 1 \times 16^{-1} + 10 \times 16^{-2} + 5 \times 16^{-3}$

$$= \frac{1}{16} + \frac{10}{256} + \frac{5}{4096}$$

$$= \frac{421}{4096}$$

$$= (0.10278)_{10}$$

Therefore, from the above calculations we can write that $(3FA.1A5)_{16} = (1018.10278)_{10}$

You do: $(1FA.2A)_{16} = (?)_{10}$

Decimal to Hexadecimal Conversion:

For Integer Number
To convert any decimal number to its hexadecimal equivalent you have divide the number by the base i.e. 16 and save the remainder until the dividend/quotient reaches 0. At each division, the remainder provides a digit of the converted number, starting with the least significant digit (LSD) and the last remainder is the most significant digit (MSD). The hexadecimal number is formed with the help of remainder numbers starting from MSD to LSD.

For Example: Convert $(423)_{10}$ to Hexadecimal

Number/Base	Quotient	Remainder
423/16	26	7
26/16	1	10 = A
1/16	0	1

Therefore, $(423)_{10} = (1A7)_{16}$

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For Fraction Number
To convert any fractional decimal number to hexadecimal number, multiply the number with the base of hexadecimal number (i.e. 16) successively and record the integer parts, moving away from the decimal point to the right.
For example, Convert (0.0628)10 decimal fraction to hexadecimal.

Number	Base	Product	Integer	Fraction
0.0628	16	1.0048	1	0.0048
0.0048	16	0.0768	0	0.0768
0.0768	16	1.2288	1	0.2288
0.2288	16	3.6608	3	0.6608
0.6608	16	10.5728	10 = A	0.5728

Therefore, $(0.0628)_{10} = (0.1013A....)_{16}$

Binary Coded Decimal Numbers (BCD)

In this system, numbers are represented in a decimal form, however each decimal digit is encoded using a four bit binary number. For example: The decimal number 136 would be represented in BCD as follows:

136 = 0001 0011 0110
1 3 6

Conversion of numbers between decimal and BCD is quite simple. To convert from decimal to BCD, simply write down the four bit binary pattern for each decimal digit. To convert from BCD to decimal, divide the number into groups of 4 bits and write down the corresponding decimal digit for each 4 bit group.